

Beliefs and 'Believes'

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It would be a convenience, nothing more, if we could take the propositional content of a sentence in context as its semantic value. But we cannot.

Lewis (1980)

Introduction

I take it that propositions at least some of the following (and possibly other) things: the meanings of sentences, the things we believe, assert, doubt, etc., and the referents of ‘that’-clauses. A common account concerning propositions is that they are sets of worlds, namely, the set of worlds in which the proposition is true. For instance, the proposition “Grass is green” denotes a set of worlds, namely the set of all the worlds in which grass is green.¹ Relatedly, attitude reports are sentences that report a certain cognitive relation of an individual towards a proposition. For example, “Claudia believes that grass is green” expresses a cognitive relation between Claudia and the proposition that grass is green, namely, a believing-relation. Similar attitude verbs are ‘asserts’, ‘knows’, ‘doubts’, etc. A common semantics for attitude reports is, what Bach (1997) calls, the *relational analysis of attitude reports*. This analysis suggests that attitude reports have the logical form they appear to have, that is, they express a relation between an individual and a proposition. Crucially, the semantic value of the embedded ‘that’-clause *is* the relevant proposition. For example, Soames (1988, pp. 105-106) says, “attitudes are relations to propositions ... To believe that ‘S’ is to believe the proposition that S.” The underlying idea seems to be the following: attitude reports relate an agent, α , to a proposition, ϕ , by sentences of the form $\ulcorner \alpha \text{ } v\text{ 's } \phi \urcorner$, therefore the semantic value of ‘ ϕ ’ must be the proposition that α has an attitude towards. Bach (1997, p. 222) dubs this *propositionalism*—i.e. “[t]he semantic value of a ‘that’-clause is a proposition.”² Even

¹For a ‘sets of worlds’-account of propositions and an account of what propositions are see Stalnaker (1976).

²Rabern (2012a, p. 81) introduces a more general version of propositionalism, namely, what he calls the *identification thesis*, that is the semantic value of a sentence ϕ is the proposition that one believes when one believes that ϕ . This more general thesis is related to the Dummettian distinction to be discussed later on in this paper and I will often use it to refer to the more specific propositionalism.

though this position seems very natural, I believe that it is ultimately mistaken. In this paper I focus on Soames' (1987; 2008) very influential argument, which is based on a relational analysis of attitude reports in combination with propositionalism, against unstructured propositions, i.e. propositions as sets of truth-supporting circumstances.³

There are a variety of problems for 'sets of truth-supporting circumstances' accounts of propositions; for example, the logical omniscience problem. The problem being that logical necessities, such as ' $2 + 2 = 4$ ' and ' $x^{\frac{1}{2}} = \sqrt{x}$ ', are true in exactly the same set of worlds, namely the set of all possible worlds. Therefore, if you have a belief with regards to one of these, you have a belief with regards to *all* of these. That is, on the relational analysis of attitude reports an agent believes a proposition, which in turn is a set of worlds; so if you have a belief towards one of these you should have a belief towards any proposition that is equivalent to that set of worlds, i.e. all logical necessities. Even though there are ways to avoid these problems, for example allowing for finer grained circumstances (e.g., incomplete circumstances consisting of only one mathematical proposition), Soames claims that his argument runs deeper and is aimed at sets of truth-supporting circumstances no matter how fine grained.

I argue that his argument relies on contentious principles concerning the interaction of attitude operators with free variables embedded in their scope. I suggest that the most promising way of challenging this is to appeal to the distinction between the objects of our attitudes and the semantics of attitude reports. Accordingly, this paper can be seen as an attempt to avoid a very influential argument against the view that the objects of belief are sets of truth-supporting circumstances (i.e. Soames' (1987; 2008) argument).

In the first section, I explicate Soames' argument, which takes form as a *reductio ad absurdum*, and show that a particular version can get off the ground with fewer, and slightly weaker, assumptions. Moreover, I fully explicate the details of the derivation that leads to the absurd conclusion. In the second section, I discuss three possible ways of resisting Soames' conclusion and elaborate on one of these, namely, one that focuses on the semantics for attitude reports. After showing how this new

³Throughout this paper I will use 'unstructured propositions' and 'propositions as truth-supporting circumstances' interchangeably. Furthermore, a 'sets of worlds'-account of propositions is a type of truth-supporting circumstances/unstructured propositions account.

semantics might block the argument, I suggest that this account might in fact be interpreted in support of Soames’ conclusion. In the third, and final, section, I argue that there is a way to interpret the particular semantics for attitude reports as an argument against Soames’ *reductio*. This interpretation is related to Dummett’s (1973) distinction between semantic value and assertoric content. In light of this, I suggest that it may be the case that Soames’ argument is based on a conflation of these two.

First, some notational conventions I use throughout this paper. Expressions occurring within double brackets ($\llbracket \cdot \rrbracket$) denote their extensions, for example, $\llbracket \text{‘Benjamin Franklin’} \rrbracket$ denotes the man Benjamin Franklin. To avoid unnecessary cluttering, I will suppress the quotation marks that indicate mention, instead of use, within such double brackets. For the purposes of this paper the only relevant parameters, relative to which expressions denote their extension, are the context, c , an assignment function, g , and the world-parameter, w . I will therefore suppress any other coordinates on most occasions and simply formulate denotations as follows: $\llbracket \phi \rrbracket^{c,g,w}$.

1 Against Unstructured Propositions

In this section I will set out Soames’ (1987; 2008) argument against unstructured propositions. I will explicate the starting assumptions and give formal regimentations of them. I suggest that by using these particular formal regimentations one could get the “variables” version of the argument off the ground with fewer and slightly weakened assumptions. After explicating and formalising these, I will briefly flag some of the assumptions that one could object to (I will get back to those in the next section). Finally, I will crunch through the semantics and derivation that leads to Soames’ conclusion.

In his original article, Soames goes through a variety of arguments against unstructured propositions. Importantly, his main argument is aimed at propositions as sets of truth-supporting circumstances “no matter how fine-grained” we take these circumstances to be (Soames, 1987, p. 52). Note that I use the term *fine grained* here, and throughout this paper, in the most neutral way possible. All I intend to convey is that if P is finer grained than Q , it means that P is able to capture distinctions that the coarser grained Q cannot.⁴ Soames sets out three different ver-

⁴I mention this as the term *fine grained* is a somewhat charged term. For example, Pickel (2013)

sions of his main argument, all of which concern the ancients’ beliefs with regards to Hesperus and Phosphorus.⁵ The different versions use either the names, i.e. ‘Hesperus’ and ‘Phosphorus’, demonstratives, e.g. ‘this’ and ‘that’, or variables, e.g. ‘ x ’ and ‘ y ’, but essentially they all come down to the same argument. I will focus on Soames’ main argument concerning variables, which may teach us some interesting lessons for the general version of Soames’ argument.⁶

1.1 The Starting Assumptions

The argument takes form as a *reductio ad absurdum* based on seemingly innocent assumptions. I have changed the notation of the assumptions slightly in compliance with the notational conventions used throughout the paper; these changes do not influence the assumptions or argument in any way.⁷ To get the argument off the ground, all that is needed are some simple satisfaction conditions and a semantic clause for ‘believe’ (as Soames, I will suppress the notion of a model here).⁸ First of all, we need the simple semantic clauses for conjunction, predication, and existential quantification;

$$(C) \llbracket \phi \wedge \psi \rrbracket^{c,g,w} = 1 \text{ iff } \llbracket \phi \rrbracket^{c,g,w} = 1 \text{ and } \llbracket \psi \rrbracket^{c,g,w} = 1$$

$$(P) \llbracket R(\alpha_1, \dots, \alpha_n) \rrbracket^{c,g,w} = 1 \text{ iff } \langle g(\alpha_1), \dots, g(\alpha_n) \rangle \in I(R, w)$$

$$(E) \llbracket \exists \alpha \phi \rrbracket^{c,g,w} = 1 \text{ iff there is an assignment function } g' \text{ such that } g'[\alpha]g \text{ and } \llbracket \phi \rrbracket^{c,g',w} = 1$$

argued that Cumming’s use of ‘fine grained’ wrongfully suggests that, from any level of fine grain, one could reconstruct the coarser grain levels. I do not want to engage in this discussion here, all I intend to convey is that a level of fine grain can capture certain distinctions that coarser grained level cannot.

⁵These concern the classic example that the ancients named the brightest star in the morning ‘Phosphorus’ and the brightest star in the evening ‘Hesperus’, not knowing that, as was discovered later, they named the same object, namely Venus, twice.

⁶Soames (2008) focuses on the argument concerning names as these are “simple, and easy to understand.” However, variables seem to be at the heart of the problem, Soames says “variables alone [...] would be sufficient” (2008, p. 269).

⁷See Soames (1987, pp. 48-51) and (2008, pp. 267-268) for the original notation.

⁸For these particular clauses a model would contain non-empty sets of contexts, worlds, and objects, an interpretation function, and an accessibility relation— $M = \langle C, W, D, I, \mathcal{B} \rangle$ —where \mathcal{B} is an accessibility relation that holds between an individual-world pair and all the worlds that are compatible with the individual’s beliefs at that world.

In the last clause, ‘ $g'[\alpha]g'$ ’ should be read as ‘an assignment function g' that differs at most from g in its assignment to α ’. Secondly, we need a semantic clause for attitude reports. Soames (1987, p. 49) states that “[p]ropositional attitude sentences report relations to the semantic contents of their complements,” which can be construed as follows:

$$(B) \llbracket \text{BEL}_\alpha \phi \rrbracket^{c,g,w} = 1 \text{ iff } \forall w', \text{ if } w' \in \mathcal{B}(\llbracket \alpha \rrbracket^{c,g,w}, w), \text{ then } \llbracket \phi \rrbracket^{c,g,w'} = 1$$

(B) states that ‘ α believes that ϕ ’ is true if and only if in all the worlds w' , compatible with α ’s beliefs in w ($\mathcal{B}(\alpha, w)$), ϕ is true.⁹ Finally, the satisfaction conditions for atomic variables:

$$(V) \llbracket \alpha \rrbracket^{c,g,w} = g(\alpha)$$

(V) entails that variables are *rigid* and *directly referential*—i.e. they have the same value in all worlds and that their “semantic content relative to a context (and assignment of values to variables) is [their] referent relative to the context (and assignment)” (Soames, 1987, p. 50).

Importantly, formalising the assumptions in this way makes that Soames’ (restricted) compositionality principle (1987, p. 66, footnote 5) no longer needs to be explicitly stated. Soames states a compositionality principle where he mentions that the content of complex expressions is constructed out of the contents of its constituents. However, all that is compositionally needed for the argument is embodied in the individual satisfaction conditions. Similarly, Soames (1987, p. 50, principle **A3**) provides a principle for the distribution over conjunction of propositional attitude verbs (e.g., if α believes that ϕ and ψ , then α believes that ϕ and α believes that ψ). This also does not need to be explicitly stated anymore as it is entailed by the semantics for conjunction and ‘believe’. Finally, note that these assumptions are slightly weakened as I will be focussing on the argument concerning variables;

⁹This is similar to Hintikka’s (1969) semantics for attitudes. The common Hintikka-style semantics for attitude reports are (based on Von Fintel and Heim (2011) description) as follows:

$$\cdot \llbracket \text{believe} \rrbracket^{c,g,w} = \lambda p_{(s,t)} \cdot \lambda x \cdot \forall w' : wR_x^{\mathcal{B}} w' \rightarrow \llbracket \phi \rrbracket^{c,g,w'} = 1$$

Where $\lceil wR_x^{\mathcal{B}} w' \rceil$ is a relation between all w' compatible with x ’s beliefs in w and ‘ $\lambda p_{(s,t)}$ ’ is a function of intensions.

it might very well be that Soames’ original assumptions are needed for a different version of the argument, e.g. concerning names.

Before turning to the derivation that leads to the conclusion that unstructured propositions cannot be the objects of our beliefs, it is worth pointing out some assumptions that are made in the satisfaction conditions stated above to which one could object. Obviously, one could, following Soames, challenge the assumption that propositions are sets of truth-supporting circumstances and propose any of a range of structuralist accounts (see Ripley 2012, secs. 1.2.2 and 2 for possible structuralist accounts). However, that would defeat the purpose of this paper—i.e. defending an account of unstructured propositions. Below are three possible assumptions one could challenge *without* giving up the unstructured propositions account; challenging any of these assumptions could potentially lead to denying the absurd conclusion. I will discuss these more elaborately, and some of the consequences for each of them, in section 2.

First of all, as mentioned before, the semantics for variables presented above entail that variables are rigid *de jure*—i.e. they are not even sensitive to the world-parameter.¹⁰ However, on a Carnapian (1956, §10) account, variables are no longer rigid, as they can have non-constant intensions.¹¹ That is, variables at an assignment can still differ in value per world—i.e. they *are* sensitive to the world-parameter. Elbourne (2010), for example, uses similar considerations to argue against Soames. Secondly, Ripley (2012) would challenge a whole range of assumptions in the satisfaction conditions given above. For example, the circumstantialist semantics presented by Ripley would not verify (C) and (E). However, below I will focus more on a point that circumstantialists in general might press: the allowing of impossible circumstances.

Finally, the semantics for ‘believe’ that is used assumes that an attitude verb shifts *only* the world-parameter. Essentially, the semantics assumes that attitude verbs are restricted universal quantifiers over worlds in attitude-spaces (for example, quantifying over the worlds in one’s belief space). Even though this is a common semantics for attitude reports, not everyone would agree. For example, Israel and Perry (1996) suggest that attitude verbs are *monstrous*—i.e. they shift some param-

¹⁰I will go into the distinction between rigidity *de jure* and *de facto* in more detail in section 2.1.

¹¹Carnap used the terms ‘value extensions’ and ‘value intension’ for variables, I will suppress the use of ‘value’.

eter of the context. Following a more recent defence of such a view, i.e. Schlenker (2003), a number of related views suggest that attitude verbs shift the variable assignment function (e.g., Cumming 2008, Ninan 2010, Santorio 2012, Pickel 2013, etc.). I will refer to these latter views, generalising over what they have in common, as *assignment-shifting* views.

Before we start pressing these assumptions, let us go through the *reductio* in full detail.

1.2 Explicating the Derivation

What follows will differ slightly from Soames’ original presentation (1987, pp. 60–61), however, these differences do not affect the argument in any significant way. The *reductio* takes as its starting point two widely accepted, pre-theoretic facts;

R1 There is a planet x that is seen in the morning sky and a planet y that is seen in the evening sky and the ancients believed that x was seen in the morning and y was seen in the evening

$$\exists x \exists y (Mx \wedge Ey \wedge \text{BEL}_\alpha(Mx \wedge Ey))^{12}$$

R2 The planet seen in the morning is the planet seen in the evening

$$\exists x \exists y (\forall z (Mz \leftrightarrow z = x) \wedge \forall z (Ez \leftrightarrow z = y) \wedge x = y)^{13}$$

These two innocent facts, combined with the satisfaction conditions presented above, lead to the following conclusion:

Con. The ancients believed that there was a planet that was seen both in the morning and in the evening sky

$$\text{BEL}_\alpha(\exists z (Mz \wedge Ez))$$

To see how Soames uses the assumptions to derive the conclusion, let us crunch through the semantics and derivation, explicating what assumptions are at play

¹²Here, and throughout this paper, ‘M’ and ‘E’ are, respectively, the predicates: ‘A planet that is seen in the morning sky’ and ‘A planet seen in the evening sky’. ‘ BEL_α ’ is read as ‘the ancients believed that...’.

¹³Soames (1987, p. 60) formalises **R2** as follows, “(the $x : Mx$) = (the $y : Ey$)”. However, to avoid complicating our semantics with clauses for definite descriptions, these are ‘Russelled’ out.

where. First of all, note that a combination of **(R1-R2)** could be rewritten—i.e. from **(R1-R2)** we can conclude the following:

$$\mathbf{R1:} \quad \exists x \exists y (Mx \wedge Ey \wedge \text{BEL}_\alpha(Mx \wedge Ey))$$

$$\mathbf{R2:} \quad \exists x \exists y (\forall z (Mz \leftrightarrow z = x) \wedge \forall z (Ez \leftrightarrow z = y) \wedge x = y)$$

$$\therefore \quad \exists x \exists y (x = y \wedge \text{BEL}_\alpha(Mx \wedge Ey))$$

This can be quite easily proven in simple first-order logic. First apply existential instantiation on both **(R1-R2)**, after which universal instantiation is applied twice in order to get something of the form ‘ $c = a \wedge d = a$ ’. Given transitivity, it is a very simple reconstruction from this to the conclusion (see the Appendix for a complete derivation). Going through the semantics of this intermediate conclusion will get us all that we need to start the derivation.

$$\llbracket \exists x \exists y (x = y \wedge \text{BEL}_\alpha(Mx \wedge Ey)) \rrbracket^{c,g,w} = 1 \text{ iff}$$

there is an assignment function $g'[x, y]g$ such that¹⁴

$$\llbracket x = y \wedge \text{BEL}_\alpha(Mx \wedge Ey) \rrbracket^{c,g',w} = 1, \text{ which is the case iff}$$

$$\llbracket x = y \rrbracket^{c,g',w} = 1 \text{ and } \llbracket \text{BEL}_\alpha(Mx \wedge Ey) \rrbracket^{c,g',w} = 1$$

It is of great importance that the first conjunct, in combination with **(V)**, entails the following:

$$\llbracket x = y \rrbracket^{c,g',w} = 1 \text{ iff } \llbracket x \rrbracket^{c,g',w} = \llbracket y \rrbracket^{c,g',w}$$

And, as variables are *not* sensitive to the world-parameter, it follows that;

$$\mathbf{(R)} \quad \llbracket x \rrbracket^{c,g',w} = \llbracket y \rrbracket^{c,g',w} \rightarrow \forall w' (\llbracket x \rrbracket^{c,g',w'} = \llbracket y \rrbracket^{c,g',w'})$$

Equivalently, in all the worlds where ‘ x ’ is F , ‘ y ’ is also F . Essentially, this is a formalisation of Kripke’s (1980) definition of rigidity: “[l]et’s call something a *rigid designator* if in every possible world it designates the same object” (p. 48, original emphasis). As this will be of great importance in the derivation, let us label this **(R)**, for rigidity.

¹⁴The notation ‘ $g'[x, y]g$ ’ is a simplification of ‘there is an assignment $g'[x]g$ such that ... and there is an assignment $g''[y]g'$ such that ...’ to avoid cluttering.

With all this preamble in place, it is important to get clear on the objective of the argument. We take ' $\llbracket \text{BEL}_\alpha(Mx \wedge Ey) \rrbracket^{c,g,w} = 1$ ' as our starting point and aim to show that, given this, ' $\llbracket \text{BEL}_\alpha(\exists z(Mz \wedge Ez)) \rrbracket^{c,g,w}$ ' is also true.

Given: $\llbracket \text{BEL}_\alpha(Mx \wedge Ey) \rrbracket^{c,g,w} = 1$ and $\llbracket x = y \rrbracket^{c,g,w} = 1$;

Show: $\llbracket \text{BEL}_\alpha(\exists z(Mz \wedge Ez)) \rrbracket^{c,g,w} = 1$

So, let us crunch through the derivation that leads to the absurd conclusion (**Con**);¹⁵

Step 1: From $\llbracket \text{BEL}_\alpha(Mx \wedge Ey) \rrbracket^{c,g,w} = 1$ and (**B**) we get:

(i) $\forall w'$, if $w' \in \mathcal{B}_\alpha$, then $\llbracket Mx \wedge Ey \rrbracket^{c,g,w'} = 1$

Step 2: By applying distribution of conjuncts (**C**) to (i) we get:

(ii) $\forall w'$, if $w' \in \mathcal{B}_\alpha$, then $\llbracket Mx \rrbracket^{c,g,w'} = 1$ and $\llbracket Ey \rrbracket^{c,g,w'} = 1$

Step 3: Given the semantics for predicate sentences (**P**), it follows that:

(iii) $\forall w'$, if $w' \in \mathcal{B}_\alpha$, then $\llbracket x \rrbracket^{c,g,w'} \in I(M, w')$

Step 4: From (iii) and the rigidity of variables (**R**), we get:

(iv) $\forall w'$, if $w' \in \mathcal{B}_\alpha$, then $\llbracket y \rrbracket^{c,g,w'} \in I(M, w')$

Step 5: Applying the semantics of predicate sentences (**P**) to (iv), gets:

(v) $\forall w'$, if $w' \in \mathcal{B}_\alpha$, then $\llbracket My \rrbracket^{c,g,w'} = 1$

Step 6: From (ii), (v), and the semantics for conjunction (**C**), we get:

(vi) $\forall w'$, if $w' \in \mathcal{B}_\alpha$, then $\llbracket My \wedge Ey \rrbracket^{c,g,w'} = 1$

Step 7: Finally, from (vi) and existential generalisation (**E**):

(vii) $\forall w'$, if $w' \in \mathcal{B}_\alpha$, then $\llbracket \exists z(Mz \wedge Ez) \rrbracket^{c,g,w'} = 1$

Step 8: Thus, applying the semantics for 'believe' to (vii):

(\therefore) $\llbracket \text{BEL}_\alpha(\exists z(Mz \wedge Ez)) \rrbracket^{c,g,w} = 1$

What is crucial are the assumptions that attitude verbs do not shift the assignment function and that variable assignments are not sensitive to the world-parameter.¹⁶

¹⁵For a complete derivation from ' $\exists x \exists y(x = y \wedge \text{BEL}_\alpha(Mx \wedge Ey))$ ' to ' $\text{BEL}_\alpha(\exists z(Mz \wedge Ez))$ ', see the Appendix.

¹⁶Thanks to Bryan Pickel (p.c.) for pointing this out and wording it this way.

That is a completely worked out version of Soames' argument against propositions as of truth-supporting circumstances. Given the seemingly innocent satisfaction conditions and two pre-theoretic facts about the ancients' beliefs about Hesperus and Phosphorus, we are able to derive an absurd conclusion, namely, that the ancients believed that there was one object that was both seen in the morning sky and the evening sky.

2 Rigidity, Impossible Worlds, and Assignment Shifting

As I mentioned in section 1.1, the satisfaction conditions presented above for the variable version of Soames' argument contain some assumptions that one might disagree with. If one wants to resist Soames' conclusion, yet maintain propositions as sets of truth-supporting circumstances, it would have to be by challenging one of those assumptions.

In this section I will briefly explore the assumptions that both a Carnapian or an impossible world theorist might press and the consequences these positions have. Finally, I turn to the assumption the assignment-shifters would challenge, for whom I will use Cumming's (2008) account as a representative. According to them, attitude verbs do not only shift the world-parameter, but also the assignment function. This means that the semantics for attitude verbs, (**B**), would have to be changed to accommodate this.

2.1 Carnap, Ripley, and Changing Worlds

Carnapian value intensions

Before we go into the Carnapian object to Soames' assumptions, let us briefly go over a distinction that is important for such an objection. Kripke (1980, p. 21, footnote 21) noted that there are two distinct 'forms' of rigidity and labelled these rigidity *de jure* and rigidity *de facto*. The former refers to terms that are rigid *by stipulation*, that is, such a term is rigid by definition or law (e.g., names on Kripke's account) . On the other hand, terms that are rigid *de facto* merely turn out to designate one and the same unique object after the term is evaluated at all possible worlds. For example, 'the smallest prime' is rigid *de facto* as it turns out that it

designates the number two in all worlds. Importantly, on the semantics presented above, variables are rigid *de jure*. However, according to Carnap (1956, § 10) they are not. To see why variables are not rigid on a Carnapian semantics we have to go over the particular extensions and intensions variables have according to Carnap and how these behave in modal contexts.

On a Carnapian semantics, individual variables (for example, the variables used in Soames’ argument) have individuals as their extensions and *individual concepts* as their intensions. The distinction between individuals and individual concepts is best shown by the following example. The expressions ‘Walter Scott’ and ‘the author of Waverly’ both have the same extension, namely the individual Sir Walter Scott, however, they differ in their intensions—i.e. the associated individual concepts. The individual concept of ‘Walter Scott’ is a function from worlds to extensions, which one might think to be a constant function to the person Sir Walter Scott.¹⁷ The individual concept of ‘the author of Waverly’, on the other hand, is a function from worlds to extensions, which is a non-constant function to whomever wrote the Waverly novels in that world. Carnap is very explicit about the behaviour of individual variables under modal operators. He says, “I think that individual variables in modal sentences [. . .] must be interpreted as referring, not to individuals, but to individual concepts” (1956, p. 180). Similarly, Elbourne (2010, p. 104) notes that a great number of semanticists have suggested that singular terms refer to individual concepts instead of to individuals.

The important thing is that on a Carnapian account variables can have non-constant intensions, that is, they are no longer rigid. Hence, the rigidity thesis that was explicitly stated in the assumptions above (**R**) becomes invalid;

$$\llbracket x \rrbracket^{c,g,w} = \llbracket y \rrbracket^{c,g,w} \not\rightarrow \forall w' (\llbracket x \rrbracket^{c,g,w'} = \llbracket y \rrbracket^{c,g,w'})$$

This means that on a Carnapian semantics the following expressions would be consistent (whereas for Soames, they would not):

$$\begin{aligned} & \exists x \exists y (x = y \wedge \neg \Box (x = y)) \\ & \exists x \exists y (x = y \wedge \neg \text{BEL}_\alpha (x = y)) \end{aligned}$$

¹⁷Carnap would use ‘state-descriptions’ where I say ‘worlds’, I use the latter for the continuity of terms.

Denying **(R)** blocks the move from (iii) to (iv) in the *reductio* derivation, preventing the conclusion. Recently, Elbourne (2010) objected to Soames’ argument on similar grounds. In sum, Elbourne criticises Soames “for assuming that various singular terms are directly referential when in fact there are good grounds to doubt this” (p. 101).

There are multiple consequences of such a Carnapian account of variables, I will focus on the one that combines least well with Soames’ views. Rigidity is motivated by issues Quantified Modal Logic (henceforth, QML) in connection with quantification and variables, for example, rigidity is needed if one wants to maintain an ‘objectual’ interpretation of quantification (see Stanley 1997, sec. II). If we allow variables to be non-rigid, we would need a restricted universal instantiation for rigid terms to maintain the following:¹⁸

$$\forall x\forall y(x = y \rightarrow (\phi_x \leftrightarrow \psi_y))$$

One may argue that introducing such a restricted universal instantiation rule is ‘unappealing’. As Stanley (1997, p. 566) says,

If one wishes to preserve classical substitution, as well as the objectual conception of satisfaction, then one must ensure that one’s variables [. . .] are rigid.

Soames, for example, would be strongly opposed to losing the rigidity of variables, he echoes Kaplan in various places claiming that “[v]ariables are the paradigm examples of [directly referential] terms” (1987, p. 50).

To recap, one might doubt the rigidity or “direct reference” assumption for variables and therefore claim that the conclusion to draw from Soames’ argument is that we have to dispense of **(V)** instead of the thesis that propositions are sets of truth-supporting circumstances. However, as we have seen there are challenges stemming from QML and a wide range of arguments from Kripke (1980) that suggest that variables *are* directly referential. Furthermore, Soames would simply disagree with the fact that variables are not directly referential, leaving the philosophical dialogue at a deadlock.

¹⁸Such that ‘ ϕ_x ’ and ‘ ψ_y ’ are both open formulae that differ only in that ‘ ϕ ’ contains a free variable ‘ x ’ where ‘ ψ ’ contains a free variable ‘ y ’.

The following two options (that is, impossible world theories or assignment-shifting accounts of attitude verbs) are, *prima facie*, less in conflict with Soames’ view and allow us to maintain more of the original assumptions. I will argue that we should ultimately go for the second option—i.e. that attitude verbs shift the assignment function.

Ripley’s fine-graining propositions

Ripley (2012, sec. 3.1) presents a very elaborate circumstantialist semantics to deal with a variety of problems for unstructured propositions. His semantics would reject multiple assumptions in the satisfaction conditions presented above (see Ripley 2012, p. 115), however, I will focus on a slightly more general objection. That is, an objection that impossible world theorists may make independent of Ripley’s particular semantics, namely, allowing for impossible circumstances.¹⁹

Ripley explains that a particular way of fine-graining propositions is not to think of them as sets of possible worlds, but as sets of *circumstances*. Circumstances, as opposed to possible worlds, may be incomplete or impossible (e.g., consist out of only one proposition or impossible propositions). For example, if propositions are sets of possible worlds, we would not be able to account for the difference in truth-value of sentence such as (1a-1b):²⁰

- (1) a. If Amy squared a circle, Amy would become famous.
 b. If Sarkozy squared a circle, Amy would become famous.

As that the antecedents of both (1a-1b) refer to the empty set, a ‘sets of possible worlds’ account would have to assign the same truth-value to both. Hence, to account for the intuitive difference between (1a-1b) Ripley allows for impossible circumstances.

Following the difference between circumstances and possible worlds, Ripley makes the following distinction;

Shmidentity: Equivalence that holds in all possible worlds

Identity: Equivalence that holds in all possible circumstances

¹⁹ For instance, objections made by Edelberg (1994) against Soames are in a similar vein as the ones presented below.

²⁰These are Ripley’s examples (7-8), (2012, p. 99).

Thus, if ‘identity’ on Ripley’s account means equivalence across all circumstances, Hesperus is no longer identical to Phosphorus. When applying this distinction to Soames’ argument, Ripley says,

[i]mportantly, [Soames’] argument depends on the premise that Hesperus = Phosphorus. It is not enough for the argument to go through that Hesperus is *shmidentical* to Phosphorus. The premise must be that they are *identical*. This premise, though, is rejected by the present circumstantialist theory. (2012, p. 116)

Allowing for impossible circumstances thus blocks the initial assumption (**R2**), thereby preventing the argument from even getting off the ground. So, even though Soames (2008, p. 267) claims that his argument argues against the view of propositions as sets of truth-supporting circumstances, “no matter how fine-grained the circumstances are taken to be,” Ripley claims that the distinction between *shmidentity* and identity does block the argument.

Ripley leaves us with two possible ways of interpreting his account. Firstly, one may interpret his account as claiming that Hesperus is no longer identical to Phosphorus, they are merely ‘shmidentical’. This leaves us with the question which equivalence claims are identical and which ones are *shmidentical*. The second interpretation is that Ripley suggests that we have been using the term ‘identity’ incorrectly, what we really mean is ‘shmidentity’. This seems to keep all equivalence claims on a par (namely, they are all ‘shmidentity’), yet has some radical consequences. For example, Ripley would reject a common semantics for identity statements;

$$\llbracket \phi = \psi \rrbracket^{c,g,w} = 1 \text{ iff } \llbracket \phi \rrbracket^{c,g,w} = \llbracket \psi \rrbracket^{c,g,w}$$

On Ripley’s account one could hold that there are circumstances w' such that $\llbracket \phi = \psi \rrbracket^{c,g,w'} = 1$, while $\llbracket \phi \rrbracket^{c,g,w'} \neq \llbracket \psi \rrbracket^{c,g,w'}$. Similarly, he seems to have to deny transitivity—i.e. on his account x can be identical to y and y can be identical to z , without x being identical to z .²¹

²¹Note that Ripley uses ‘ \approx ’ for *shmidentity* and ‘=’ for identity, strictly speaking then Ripley only gives up these two things if the semantics would be formulated using ‘ \approx ’ instead of ‘=’—i.e. $\llbracket \phi \approx \psi \rrbracket^{c,g,w} = 1$, while $\llbracket \phi \rrbracket^{c,g,w} \neq \llbracket \psi \rrbracket^{c,g,w}$.

Even though some might think that the benefits outweigh these consequences, I will now turn to an account that I think can achieve similar benefits, without these consequences.

2.2 Attitude Reports and Assignment-Shifting

As I mentioned above, there have been accounts that diverge from the common semantics for attitude reports that Soames uses. On such accounts attitude verbs are often *monsters*, i.e. they shift a parameter of the context. In line with this, there recently have been views developed where the attitude reports shift specifically the assignment function (e.g., Cumming 2008, Ninan 2010, Santorio 2012, Pickel 2013).²² In this section I will focus on Cumming’s account according to which attitude verbs operate on something more fine grained than sets of worlds, namely sets of world-assignment pairs. As we will see below, word-assignment pairs allow for distinctions that sets of worlds cannot make. Accordingly, the semantic clause for ‘believe’ (**B**) has to be adjusted as operating on world-assignment pairs instead of sets of worlds.

Open- and Closed Propositions

The assignment-sensitivity of attitude verbs is based on an independently motivated argument concerning the semantics of variables. And so, to fully understand the distinction that follows, let us briefly go over the semantics for variables:

- For a variable α , $\llbracket \alpha \rrbracket^{c,g,w} = g(\alpha)$
- $\llbracket \forall \alpha \phi \rrbracket^{c,g,w} = 1$ iff for all $i \in D$, $\llbracket \phi \rrbracket^{c,g[\alpha:=i],w} = 1$

As mentioned earlier, this semantics implies that variables are rigid—i.e. they are not sensitive to the world-parameter. The semantics for the (universal) quantifier states that a formula containing a universally bound variable is true at an assignment g iff for all the assignments g' , ‘ ϕ ’ is true at g' and g' differs at most from g in that for all $i \in D$ it assigns i to α . That is, the universal quantifier ‘looks’ across all assignment functions at the assigned value of the embedded variable (or formula containing it) and is true if the embedded formula is true at all assignments. Thus,

²²See Rabern (2013) for a clear exposition of why and how shifting the assignment function is monstrous.

formally the universal quantifier is an integer product of the sequences of truth-values of the embedded formula.²³ So, although the assignment-saturated value of variables is important for predication, atomic variables, etc., it seems that for quantification the assignment-*unsaturated* value of variables is important. This suggests, as Pickel (2013, p. 5) says, that “quantifiers have the effect of shifting the assignment functions relative to which an embedded sentence is to be evaluated.”

This ‘dual life’ of variables, as Pickel calls it, leads to the distinction between *open propositions* and *closed propositions*, based on, respectively, the assignment-unsaturated and assignment-saturated value of variables. Open propositions are functions from world-assignment pairs to extensions (or truth-values) or functions from assignments to closed propositions. That is, closed propositions are sets of possible worlds. For example, (2) expresses an open proposition, however, after a particular assignment function assigns ‘ x_T ’ to me, (2) denotes the closed proposition that Tom is writing on propositions.

(2) x_T is writing on propositions

Although Cumming never fully explicates this formally, it seems that one could formulate open and closed propositions as follows:²⁴

OPEN PROPOSITION IN c	CLOSED PROPOSITION IN c
$\lambda g, w. \llbracket \phi \rrbracket^{c, g, w}$	$\lambda w. \llbracket \phi \rrbracket^{c, g_c, w}$

This means that open propositions are *finer* grained than sets of possible worlds (i.e., closed propositions).

With this distinction in place, Cumming claims that attitude reports “denote a relation between an individual and something *more fine grained* than a set of worlds [...] The natural choice is the open content of the subordinate clause”—i.e. the open proposition (2008, p. 545, original emphasis). This is crucial for applying such assignment-sensitive semantics to Soames’ *reductio*.

²³Accordingly, Rabern (2013, p. 397) provides the following lexical entry for ‘ $\forall\alpha$ ’;

$$\llbracket \forall\alpha \rrbracket^g = \lambda p_{\langle \gamma, t \rangle}. \prod_{i \in U} p(g[\alpha := i])$$

²⁴Note that $\lambda g, w. \llbracket \phi \rrbracket^{c, g, i}$ is sloppy notation of $\lambda g. \lambda w. \llbracket \phi \rrbracket^{c, g, i}$, this notational sloppiness is merely to avoid unnecessary cluttering and does not influence the argument in any significant way.

Attitude Reports and Open Propositions

The claim that attitude verbs operate on open propositions amounts to reformulating the semantics for ‘believe’, (**B**), as follows:²⁵

$$\mathbf{B}^* \llbracket \text{BEL}_\alpha \phi \rrbracket^{c,g,w} = 1 \text{ iff } \forall \langle g', w' \rangle, \text{ if } \langle g', w' \rangle \in \mathcal{B}(\llbracket \alpha \rrbracket^{c,g,w}, w), \text{ then } \llbracket \phi \rrbracket^{c,g',w'} = 1$$

This entails that in the derivation (i) is true relative to an assignment g , just in case (ii) is true for every world-*assignment* pair in the belief state of the ancients. Crucially, these assignments do not need to match up with the *contextually provided assignment*—i.e. the assignment that assigns all variables to their referents of that context. The contextually provided assignment is such that it assigns the same object to ‘ x ’ and ‘ y ’, namely Venus, but obviously there are assignment functions that do not assign this object to ‘ x ’, ‘ y ’, or either. That is, the assignment function that makes the belief report true might be different from the assignment function that makes ‘ $x = y$ ’ true. For example, if the ancients believe that Venus is seen in the morning sky and Mars is seen in the evening sky, the belief report would be true if all assignment-world pairs assign Venus to ‘ x ’ and Mars to ‘ y ’. However, such assignments clearly would not make the identity claim true as Venus and Mars are two different planets.

In sum, an assignment-shifting account of attitude reports seems to maintain all of Soames’ original assumptions (albeit, slightly altered), without leading to the absurd conclusion and, importantly, is independently motivated by the semantics for variables and quantification.

2.3 Some Consequences

Changing the semantics for ‘believe’, as Cumming suggests, does have some consequences. I will briefly discuss two of them before I turn to, what I take to be, the most serious concern in section 2.4.

²⁵Note that ‘**B**’ now is an accessibility relation that holds between an individual-world pair and all the *assignment-world pairs* that are compatible with the individual’s beliefs at that world. Furthermore, one may suggest that an individual’s beliefs are relative to an assignment-world pair. That is, instead of ‘ $\mathcal{B}(\llbracket \alpha \rrbracket^{c,g,w}, w)$ ’, one may use ‘ $\mathcal{B}(\llbracket \alpha \rrbracket^{c,g,w}, \langle g, w \rangle)$ ’. See for example Pickel (2013).

Shifted Assignments, Impossible Worlds, and Rigidity

First of all, one might wonder whether the solution proposed by such an assignment-shifting account is in a Carnapian vein. That is, it might seem that on such an assignment-shifting account variables are no longer rigid (nor directly referential). If, for example, in the ancients’ belief worlds ‘*y*’ gets assigned to Mars and the contextually provided assignment assigns ‘*y*’ to Venus, then it seems that ‘*y*’ is no longer rigid. As we have seen before, accepting this would lead to some challenges in QML. However, on such assignment-shifting accounts, variables *are* still rigid. Remember that **(R)** could be thought of as a formalisation of the rigidity thesis:

$$\mathbf{(R)} \quad \llbracket x \rrbracket^{c,g,w} = \llbracket y \rrbracket^{c,g,w} \rightarrow \forall w' (\llbracket x \rrbracket^{c,g,w'} = \llbracket y \rrbracket^{c,g,w'})$$

The semantics suggested on an assignment-shifting account still allows for this, thus, variables are still rigid on these accounts. What does change is that variables are no longer *super rigid*, that is, they do not have the same value at all assignment-world pairs.²⁶ So,

$$\llbracket x \rrbracket^{c,g,w} = \llbracket y \rrbracket^{c,g,w} \not\leftrightarrow \forall \langle g', w' \rangle (\llbracket x \rrbracket^{c,g',w'} = \llbracket y \rrbracket^{c,g',w'})$$

As variables are no longer super rigid in this sense, one may worry that therefore variables are no longer directly referential. And, as we have seen before, according to Soames “[v]ariables are the paradigm examples of [directly referential] terms” (1987, p. 50).

However, things might not be as bad as they seem and we can draw lessons from Rabern’s (2013) discussion of the presence of monsters in Kaplan’s *Logic of Demonstratives*. Rabern used, as the assignment-shifters, the difference between assignment-saturated and -unsaturated value of variables to argue that variable binding is a monstrous operation as it shifts a parameter from the context. According to Rabern, this is monstrous due to a clash of Kaplan’s commitment to both a compositionality principle and a direct reference principle (with regards to variables). However, since monsters (in this case attitude verbs) operate on the assignment-unsaturated value only, we may be able to hold on to some version of the direct

²⁶I use ‘super rigidity’ here informally to distinguish designating the same object at all worlds from designating the same object at all assignment-world pairs. I do not intend to refer to the mathematical usage of ‘superrigidity’.

reference thesis for variables.²⁷ That is, it seems that on the assignment-shifting accounts variables can still be said to be directly referential at the assignment-saturated value.

So, in this sense it seems that we can hold on to a particular version of the direct reference thesis for variables, namely, that the assignment-saturated value of variables is directly referential. Interestingly, Cumming is quite explicit about this when he compares the directly referentialness of assignment-saturated variables with the directly referentialness of indexicals on Kaplan’s account. He says, “I capture the intuition of direct reference for names to precisely the extent that [Kaplan] does for indexicals,” that is, they are directly referential at the level of content (2008, p. 541).²⁸ Even more so, it seems that we could even reformulate Soames’ formulation of direct reference in a similar way. Soames (1987, p. 51) claims that to be directly referential means that the “semantic content relative to a context (and assignment of values to variables) is [the] referent relative to the context (and assignment).” On an assignment-shifting account, this thesis could be held on to if we take ‘relative to an assignment of values to variables’ to be ‘the assignment-saturated value of variables’. Given what we have seen above (i.e. the fact that variables appear to have two different values) this seems a natural way to interpret the direct reference thesis for variables.

Likewise, it seems that the assignment-shifting accounts are related to the circumstantialists account, in that there are world-assignment pairs where $\llbracket \text{Hesperus} \rrbracket^{c,g,w} \neq \llbracket \text{Phosphorus} \rrbracket^{c,g,w}$. Thus, one might wonder if the assignment-shifting accounts have to accept the same consequences as the impossible world theorists (e.g. Ripley) have to. The short answer is that they do not. As we have seen before, Ripley denies the common semantics for identity, that is, on his account there are circumstances where the following claims can both be true (namely, impossible ones):

$$\llbracket \phi = \psi \rrbracket^{c,g,w} = 1 \text{ and } \llbracket \phi \rrbracket^{c,g,w} \neq \llbracket \psi \rrbracket^{c,g,w}$$

²⁷This is inspired by Rabern’s (2013, p. 400) similar move. He says, “since monsters operate on entities that are distinction [sic.] from the assertoric content of their embedded clause, we have a straightforward and powerful argument for Dummett’s ingredient sense/assertoric content distinction.”

²⁸Cumming talks about names (instead of variables) as he argues that names are semantically on a par with variables—i.e. he refers to the intuition of direct reference for variables.

Even though on assignment-shifting accounts there are world-assignment pairs where $\llbracket \text{Hesperus} \rrbracket^{c,g,w} \neq \llbracket \text{Phosphorus} \rrbracket^{c,g,w}$, they would also hold that on those world-assignment pairs $\llbracket \text{Hesperus} = \text{Phosphorus} \rrbracket^{c,g,w} \neq 1$. Similarly, Ripley’s account seems to deny transitivity of identity in some circumstances, whereas this is maintained on the assignment-shifting accounts. For example, if relative to a particular world-assignment pair $\llbracket \text{Hesperus} \rrbracket^{c,g,w} = \llbracket \text{Mars} \rrbracket^{c,g,w}$ and Mars would also be identical to the Earth’s moon, then relative to that world-assignment pair the following would also be the case: $\llbracket \text{Hesperus} \rrbracket^{c,g,w} = \llbracket \text{the Earth’s moon} \rrbracket^{c,g,w}$.

So, although there are some similarities between the assignment-shifting account and both the impossible world theorists and Carnapian responses, it seems that the assignment-shifting accounts share mostly the benefits, without many of the consequences. However, there is one worry that is, I believe, specific to the assignment-shifting accounts and which might suggest a different absurd conclusion.

2.4 Believing Open Propositions

As I mentioned above, it seems that assignment-shifting accounts can accept most of the starting assumptions without having to accept the conclusion. However, there are some things to consider.

First of all, note that Soames could retreat to a weaker claim that propositions cannot be sets of worlds (instead of the more general ‘sets of truth-supporting circumstances’). If he would make such a move, he could simply point out that the assignment-shifting accounts are in support of his argument. On an assignment-shifting account the objects of belief are sets of world-assignment pairs, they are not sets of worlds, thus supporting Soames’ weaker claim that no theory accepting all assumptions can hold on to propositions as sets of worlds. I will leave such a move by Soames for now, as I believe that the arguments in section 3 suggest that even a retreat to this weaker claim is untenable. Secondly there is a general worry that one might press, which I believe to be a serious concern, to suggest that an assignment-shifting account comes at too high a price. Namely, does an assignment-shifting account, such as Cumming’s, commit us to the view that the objects of belief are open propositions? And if so, what consequences does this have? Are the things we believe assignment-sensitive? Do propositions vary in truth-value across assignments of values to variables? It seems very unlikely that the things we believe

are assignment-sensitive, so it would be controversial if on an assignment-shifting account they are.

Cumming himself is never quite explicit about his views on the objects of belief, however, passages in his paper seem to suggest that he thinks that the objects of belief are open propositions. For instance,

one believes an open proposition o if and only if o is true at every assignment-world pair consistent with what one believes (every pair in one’s ‘belief set’). (2008, p. 545)

This seems to entail that two people believe the same thing if they believe the same open proposition, which in turn seems to suggest that the objects of our beliefs are open propositions.²⁹

If we take it that the objects of beliefs are propositions, this seems to have the consequence that propositions differ in truth-value across assignments of values to variables. Similarly, it seems to suggest that the things we believe are assignment-sensitive. Soames might point out that even though it seems that on an assignment-shifting account we can avoid the conclusion of his *reductio*, this solution comes at too high a price. It seems very unlikely that beliefs are assignment-sensitive and that propositions vary in truth-value and Soames might argue that it therefore is better to accept his positive proposal (e.g. Soames 2012).³⁰

In the next section I want to explore the possibility of accepting an assignment-shifting account to avoid Soames’ *reductio* argument, while maintaining that the objects of our beliefs are sets of worlds.

3 The Semantics and Objects of Attitudes

As we have seen in the introduction, a very common view about attitude reports is that they report a relation between an agent and a proposition. More specifically, it is generally taken that the semantic value of the embedded expression is identical to the proposition the agent has an attitude towards. However, this identification thesis has been challenged by some (see, for example, Bach 1997 and Rabern 2012a,b). In this section I will suggest that we can draw lessons from comparing the

²⁹Cumming hints at this at multiple places, see also Pickel (2013).

³⁰Thanks to Brian Rabern (p.c.) for helping me getting clear on this issue.

assignment-shifting accounts with an eternalism account of propositions that denies the identification thesis. If these accounts are similar, then this may suggest a way to accept the assignment-shifting semantics for ‘believe’, while maintaining that the objects of beliefs are sets of worlds.

3.1 Eternalism, Content, and Semantic Value

Eternalism claims that the objects of assertion and beliefs (propositions) are eternal—i.e. they have a ‘fixed and unchanging’ truth-value over time. That is, according to eternalists (3a) is an incomplete expression and is ‘time-stamped’ to the time of the context to express the eternal proposition (3b).

- (3) a. Tom is writing on propositions
 b. Tom is writing on propositions on the 28th July 2014

However, Kaplan (1989), amongst others, raised an influential argument against eternalism, known as the *operator argument*, which goes as follows. If all propositions are eternal (i.e. time-stamped at the moment of utterance, as (3b)), then it makes no sense to have temporal operators such as ‘sometimes,’ ‘tomorrow,’ etc., in our language. That is, if propositions are eternal, then “the application of a temporal operator to such a content would have no effect; the operator would be vacuous” (Kaplan, 1989, p. 504, footnote 28). Hence, propositions must not be eternal.

This goes against the common idea that eternalism about propositions “is central and fundamental to the very idea of propositions, and is part and parcel of a philosophically entrenched conception of proposition content” (Salmon, 2003, p. 112). Salmon amends Kaplan’s account to allow for eternal propositions.³¹ He argues that Kaplan drew the wrong conclusion from the operator argument. Instead of concluding that propositions are not eternal, the right conclusion to be drawn is that “temporal operators do not operate on propositions” (Salmon, 2003, p. 131). That is, Salmon argues that Kaplan conflates propositions with the objects on which temporal operators operate.

To maintain the merits of the Kaplanian theory, Salmon argues that there is an intermediate level of semantic value, one that temporal operators take as their

³¹I focus here on Salmon’s (2003) account, however, note that this is, partly, based on earlier work such as, for example, Salmon (1986).

argument. He dubs this semantic value the *content base*, which is a function from times to intensions or, similarly, from time-world pairs to extensions.³² Salmon says;

[The] notion of the content base of an expression with respect to a given context, and the resulting reconstrual of the character of an expression, impose a fourth level of semantic value, *intermediate between the level of character and the level of content*. (2003, p. 121, emphasis added)

So, on Salmon’s account the objects that temporal operators operate on (i.e. the temporally neutral content base) come apart from the objects of belief and assertion (i.e. eternal propositions).

Interestingly enough, similar suggestions have been made with regards to a variety of different problems. I.e., on multiple occasions people have suggested that some arguments seem to conflate the semantic value of expressions and the objects they denote. For example, Humberstone (1976) and Davies and Humberstone (1980) noted that certain arguments against a particular meta-ethical picture are based on semantics. They suggest that if these notions (that is, the meta-ethical content and the semantic value) come apart, the arguments could be avoided. As Davies and Humberstone put it, some objections against subjectivism

... assume the subjectivist to be aiming at renderings of ingredient sense [i.e. semantic value]; yet the philosophical interest of subjectivism would certainly survive a retreat to the weaker position in which it was assertive content only that was at issue. (1980, p. 23)

Similarly, Yalcin (2007) and Ninan (2010) distinguish between the semantics of epistemic modal clauses and the content they communicate. This is done to sidestep arguments against contextualists account of epistemic modals. Epistemic modals are restricted quantifiers ranging over epistemic possibilities—i.e. possibilities compatible with someone’s evidence.³³ Contextualists claim that the evidence in question

³²Formally, this could be regimented as follows;

CONTENT BASE IN c	PROPOSITION IN c
$\lambda t, w. \llbracket \phi \rrbracket^{c, t, w}$	$\lambda w. \llbracket \phi \rrbracket^{c, t_c, w}$

³³I gloss over many subtleties here, however, as we are concerned with the general picture, these subtleties are not important for the purposes of this paper.

should be the evidence of the agent, or the contextually relevant group of agents. However, arguments against contextualism claim that in certain situations, the contextualists semantics of epistemic modals cannot correctly predict the truth-value of embedded epistemic modals. Imagine, for example, that (4) is uttered by a third person, say Ollie;

(4) Barbara said that Susan might be at the office

Contextualists predict that what is important for the truth-value of (4) are the possibilities that are compatible with *Ollie’s* evidence. Clearly this is not the case, the truth-value of the embedded modal claim of (4) should depend on what is compatible with the evidence of Barbara, not Ollie. Distinguishing the semantics of epistemic modals and the content they communicate, as Yalcin and Ninan do, avoids this problem (see also Rabern 2012a, p. 86).

3.2 A Dummettian Distinction

All accounts that make a distinction between the semantic value of expressions and the objects they denote are reminiscent of, and many of them are based on, Dummett’s (1973) distinction between *assertoric content* and *ingredient sense*. Dummett, who originally made the distinction when noticing a failure of compositionality while discussing Frege’s equivalence thesis (see Dummett 1973, ch. 13 and Rabern 2012b, ch. 4), said;

We must distinguish [...] between knowing the meaning of a statement in the sense of grasping the content of an assertion of it, and in the sense of knowing the contribution it makes to determining the content of a complex statement in which it is a constituent: let us refer to the former simply as knowing the content of the statement, and to the latter as knowing its ingredient sense.³⁴ (1973, pp. 446-447)

Hence, following Dummett, the assertoric content is *what is said* by a sentence, what is believed or asserted, and what is true or false. The semantic value is what an

³⁴I follow Ninan (2010) and Rabern (2012a) by following Lewis (1980) in saying ‘semantic value’ where Dummett says ‘ingredient sense’ and following Dummett in saying ‘assertoric content’ where Lewis says ‘propositional content’.

expression contributes to a more complex sentence in which it is embedded. This distinction between the semantics and objects of attitudes seems to be what all the different accounts outlined above suggest.

If there are two distinct values at play, that might suggest that there are two different questions being relevant. Two questions that Yalcin (2007, p. 1006) explicates with regards to epistemic modals;

1. What is the compositional semantics of an epistemic modal clause?
2. What informational content do utterances of epistemic modal sentences communicate?

Thus, in general we might distinguish questions regarding compositional semantics from questions regarding the objects of attitudes.³⁵

Even though these questions are clearly related, it is of crucial importance not to conflate them (see, e.g., Yalcin 2007, p. 1006 for a similar warning). For example, with this distinction in mind, one might be sceptical of the relational analysis of attitude reports and the related propositionalism. As mentioned in the introduction, the idea there is that if sentences of the form ‘ α *v*’s ϕ ’ relate an agent to a proposition, then the semantic value of ‘ ϕ ’ must be the propositions that α has an attitude towards. However, distinguishing these two values (and the related questions) raises doubts about this common view. Might it be that the semantic value of a sentence is a set of world-assignment pairs, while the proposition expressed is a set of worlds?

3.3 Attitude-Shifting and the Objects of Attitudes

Let us now turn again to the assignment-shifting accounts of attitude reports. Remember that before we made this distinction, it seemed that Cumming’s semantics for ‘believe’ had dire consequences, for example, that the objects of beliefs would be assignment-sensitive. Let us see if, with the distinction made above in mind, we can accept the assignment-shifting accounts without these consequences.

³⁵For such a more general distinction see, amongst others, Ninan (2010) and Yalcin (2014). For example, Yalcin (2014, p. 19) says that “there is little motivation for theorizing under the assumption that the compositional value of a sentence relative to context is the sort of thing that also plays certain of the other key roles paradigmatically associated with the notion of content.”

To see how and if this can be done, let us compare the assignment-shifting accounts with Salmon’s (2003) account of eternal propositions and temporal operators.³⁶ First of all, note that both these accounts are structurally very similar. For example, on the eternalists’ account (3a) expresses a different proposition in different contexts (based on the contextual time), whereas for the assignment-shifters the proposition (2) expresses varies per context (based on the assignment function).

(2) x_T is writing on propositions

(3a) Tom is writing on propositions

Moreover, comparing these two accounts in their formalised form suggests that, structurally, they both come down to the same process;³⁷

OPEN PROPOSITION IN c	CLOSED PROPOSITION IN c
$\lambda g, w. \llbracket \phi \rrbracket^{c, g, w}$	$\lambda w. \llbracket \phi \rrbracket^{c, g_c, w}$
CONTENT BASE IN c	PROPOSITION IN c
$\lambda t, w. \llbracket \phi \rrbracket^{c, t, w}$	$\lambda w. \llbracket \phi \rrbracket^{c, t_c, w}$

Secondly, and most importantly, remember how Salmon (2003, p. 121) characterised the content base, as an intermediate level of semantic value “between the level of character and the level of content.” Cumming’s description of open propositions seems very similar of this when he says, “I prefer to think of open propositions as an *intermediate layer of content*” (2008, p. 542, original emphasis). Cumming, as Salmon, is referring here to open propositions being intermediate to the level of character and the level of content.

So, as the assignment-shifters distinction between assignment-saturated value and assignment-unsaturated value seems to be similar to Salmon’s distinction between propositions and content base, maybe we can draw similar conclusions. Salmon concluded that temporal operators operate on the content base, whereas the objects of assertion and belief are propositions. A parallel conclusion for an assignment-shifting account seems then that attitude verbs operate on the open propositions of

³⁶Thanks to Brian Rabern (p.c.) for directing my attention to this similarity.

³⁷For even more formal equivalence, see Rabern’s (2012a, p. 90) formalisation of Dummett’s assertoric content/semantic value distinction;

SEMANTIC VALUE IN c	ASSERTORIC CONTENT IN c
$\lambda g, t, w. \llbracket \phi \rrbracket^{c, g, t, w}$	$\lambda w. \llbracket \phi \rrbracket^{c, g_c, t_c, w}$

expressions, whereas the objects of our beliefs are the closed propositions of expressions, i.e. sets of worlds. On this interpretation of the assignment-shifting accounts, the question “what does it mean to believe an open proposition?” seems to be conflating two distinct questions. Namely, a question concerning the semantics of attitude reports and a question about the objects of beliefs. One does not believe an open proposition (nor the assignment-unsaturated value of an expression), the objects of our beliefs can still be closed propositions (i.e. the assignment-saturated value of an expression).

This way the assignment-shifting accounts can be interpreted as an argument *against* Soames’ argument. As the distinction above indicates we can hold on to closed propositions as the objects of beliefs, while accepting the assignment-shifting semantics for ‘believes’. Furthermore, if the distinction assignment-shifting accounts make is indeed similar to the assertoric content/semantic value distinction of Dummett, making this distinction seems motivated independently of their semantic preferences.³⁸

3.4 Soames, Semantic Value, and Assertoric Content

Interpreting the assignment-shifting account in such a way (that is, concerning the semantics of attitudes, while maintaining the view that propositions are sets of worlds) is similar to suggesting that Soames fails to make the distinction between assertoric content and semantic value. However, before we accuse Soames of conflating these two, it is important to see the relation between Soames’ view, his argument, and the distinction at hand.

First of all, one might wonder whether or not Soames is at all committed to make this distinction to begin with. I believe that there are good reasons to think that he is. For instance, Yalcin (2014) provides convincing arguments (both empirical and conceptual) that these notions *should* be distinguished, no matter what semantic preferences one has. In discussing Rabern’s (2012b; 2013) arguments, Yalcin mentions that the distinction between semantic value and assertoric content already seems to be needed for basic variable-binding operations. It seems that many semantic theories incorporate such variable-binding and, moreover, Soames’ (1987)

³⁸Compare the assignment-shifting distinction with Rabern’s (2012a) formalisation of assertoric content/semantic value (see footnote 37).

argument uses variable-binding explicitly. Furthermore, in recent work Soames himself seems to warn us *against* conflating the notions of content base and (eternal) propositions:

The technical demands on temporal operators tell us *nothing* about whether the propositions sentences express are time-neutral, or time-specific. That issue must be resolved independently. (2011, p. 130, original emphasis)

If the distinction between the content base and propositions is related to the distinction between assertoric content and semantic value, then it seems that Soames could also be interpreted as warning us against conflating these two notions.

Thus, it seems then that Soames *is* committed to the distinction described above, secondly, we need to assess if he fails to make it. Yalcin (2014, p. 27) points out that philosophers often talk about ‘semantic content’, in ways that tend “to blur a conceptually important distinction” between *semantic* value and assertoric *content*. Yalcin goes on to discuss Soames’ (1989, p. 394) definition of a semantic theory and points out that “[t]his conception of semantics is problematic. It underestimates the gap between the notion of compositional semantic value and the notion of content” (Yalcin, 2014, p. 30). So, even though Soames should be committed to the distinction in question, he ‘underestimates’ the gap between semantic value and assertoric content by ‘blurring’ these notions through his use of the notion ‘semantic content’.

This suggests that a valuable lesson can be drawn with regards to Soames’ argument against unstructured propositions in general. To see what lesson this is, let us repeat Soames’ argument one more time in the simplest way possible. Soames (1987; 2008) aims to provide an argument against a particular view of what the objects of our beliefs are, namely that they cannot be sets of truth-supporting circumstances. Soames argues against this particular view using a *reductio ad absurdum* based on the semantics of attitude reports. Hence, he argues against a metaphysical picture using arguments based on our semantics. However, to paraphrase Soames (2011), the technical demands on attitude verbs tell us nothing (directly) about what the objects of our beliefs are. That issue must be resolved independently.³⁹

³⁹Note that this is also why I believe that Soames’ retreat to a weaker version (as mentioned in section 2.4) is untenable.

3.5 A Metaphysical Argument

Let us assume that Soames would agree with the arguments presented above and that he agrees that one should distinguish between the semantics and metaphysics of attitudes. In response, Soames might argue that his argument remains; he might argue that the objects of belief cannot be sets of worlds, as his argument could be reformulated as a purely metaphysical argument.

If the ancients believe that Hesperus is seen in the evening sky and Phosphorus in the morning sky, the object of that belief is the set of worlds where this is the case. And, given that Hesperus is Phosphorus, in all those worlds it is also the case that Hesperus is seen in the evening sky and that *Phosphorus is seen in the evening sky*—i.e. the latter is the same ‘object of belief’ as the former. Therefore, one might be tempted to conclude that if the ancients believe that Hesperus is seen in the evening sky, then the ancients believe that Phosphorus is seen in the evening sky.

However, even though these would be the same objects of belief, it does *not* follow that if the ancients believe that Hesperus is seen in the evening sky, that they then also believe that Phosphorus is seen in the evening sky. There are accounts of belief where what matters for one to believe a certain content is not only the content itself, but specifically *how* one believes it or the *way* one takes it. For instance, Perry (1979) distinguishes “the belief state one is in” from “what one thereby believes” (p. 18) and, on a somewhat related note, Salmon (1986, especially ch. 8) distinguishes the content of belief from the ways-of-taking the content (or guises of the content). Very crudely, on such accounts an agent might believe a certain content relative to one way of taking it (or one guise), while failing to believe *that exact same content* relative to another way of taking it.⁴⁰ Thus, the ancients might believe the set of worlds where Hesperus is seen in the evening sky in a “Hesperus” guise, while failing to believe it in a “Phosphorus” guise. Moreover, on a view where attitude verbs are sensitive to assignments, while the objects of belief are sets of worlds, the assignment-

⁴⁰A similar issue comes in in discussions concerning knowledge-how and knowledge-that. Stanley and Williamson (2001) argue that knowledge-how is, similar to knowledge-that, a relation between an agent and a proposition. They suggest that the differences between the two types of knowledge ascriptions are due to *how* one relates to a proposition, which for the knowledge-how is a practical way. Furthermore, and related to section 3.1, Stanley and Williamson also suggest that the technical demands on the semantics of knowledge-how and knowledge-that attributions should not settle questions what these attributions relate an agent to. See also Cath (Forthcoming).

sensitivity might be construed as tracking “ways-of-taking” the reference relation. Cumming (2008) even says that his “treatment of attitude verbs as operators that shift the assignment tallies with the reflection that attitude ascriptions can convey things about *how an agent conceives of the reference relation*” (p. 550, original emphasis).

Thus, from the fact that the ancients believe that Hesperus is seen in the evening sky, it does not follow that the ancients believe that Phosphorus is seen in the evening sky—even if we grant that the contents are the same. Consider the analogy with Perry’s (1979) account of the essential indexical. Say that I see myself in a mirror making a mess, without realising that the person in the mirror is me; I might then believe that the guy I see in the mirror is making a mess, without believing that *I* am making a mess (even though I am the guy in the mirror and thus the propositions—the set of worlds or the Russellian proposition—expressed by the relevant embedded sentences are the same). That is, it seems that in that case I can be said to take to the same proposition in two different ways, believing the proposition in the guise of “the guy in the mirror”, without believing the same proposition in the guise of “I, myself”. Likewise, hidden-indexical theorists generally claim that something like this is the case (see, for example, Crimmins & Perry 1989 and for a variety of different hidden-indexical theories, see Bach 1997, pp. 218-219).

Bach (1997, p. 218) discusses a variety of different hidden-indexical theories and dismisses these because they seem to have to give up compositionality. A different problem Bach points out is that hidden-indexical accounts seem to treat attitude verbs as a triadic relation instead of the dyadic relation that it appears to express from a linguistic point of view. Note that both these objections of a semantic nature, however, as we have seen, these are accounts concerning a metaphysical picture. Given that the objects of belief are now distinct from the semantics, it seems that such objections are not as strong as they were under the acceptance of the identification thesis. Thus, distinguishing between assertoric content and semantic value may allow us to pursue such views without the suggested semantic consequences, thereby avoiding (or at least anticipating) Soames’ possible retreat to a purely metaphysical argument.

4 Conclusion; Semantics and Metaphysics

I critically discussed Soames’ argument and suggested that endorsing an assignment-shifting account of attitude verbs may allow us to hold all the assumptions while avoiding the absurd conclusion of the *reductio*. However, there seemed to be dire consequences with regards to the assignment-sensitivity of beliefs and propositions. I suggested a further step that allowed us to maintain that propositions are sets of worlds. This step was distinguishing between the objects of beliefs and the semantics of ‘believes’. Let me end with a concluding remark.

Distinguishing between assertoric content and semantic value is definitely not the orthodox view and Humberstone (1976) touches a nerve when he asks if this distinction should worry us.⁴¹ It seems that many philosophers do not make this distinction, because it *does* worry them that these two notions come apart. This worry, which Rabern (2012b) dubs the *mismatch worry*, links back again to the relational analysis of attitude reports. Remember, if we distinguish between semantic value and assertoric content what is important for sentences of the form ‘ α *v*’s ϕ ’ is the semantic value of ‘ ϕ ’. The worry is that the metaphysics ‘calls’ for a relation between α and a proposition, thus not between α and a semantic value.⁴²

I do believe that language and metaphysics are indeed closely related, however, I also agree with Bach (1997) that one should not immediately let considerations from one field trump decisions in the other. An example of this is the following:

One might have scruples about the objective character of moral or aesthetic values, [...], but this should not lead one to suppose that adjectives like ‘good’ or ‘beautiful’ are to be treated differently, from a semantic point of view, from ‘round’ or ‘reptilian’. (Bach, 1997, p. 218)

Similarly, we may have particular views about what the metaphysics calls for concerning attitude reports, however, this should not lead us to suppose what the attitude verbs operate on, or vice versa.

⁴¹Humberstone actually wonders whether or not it should worry us that sameness of assertoric content does not imply sameness of semantic value. This comes down to the same worry.

⁴²Interestingly, similar worries were raised against Russell’s ‘logical fictions’, which he introduced to deal with ‘incomplete symbols’. The worry, as expressed by Urmson (1971, p. 30), is that “by eliminating the descriptive expression ‘the master of Plato’ from a proposition one does not show that the master of Plato does not, or did not, exist.”

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Appendix

In this Appendix I present the fully worked out derivations used in section 1.1. First the derivation from (**R1-R2**) to the intermediate conclusion:

$$\mathbf{R1:} \quad \exists x \exists y (Mx \wedge Ey \wedge \text{BEL}_\alpha(Mx \wedge Ey))$$

$$\mathbf{R2:} \quad \exists x \exists y (\forall z (Mz \leftrightarrow z = x) \wedge \forall z (Ez \leftrightarrow z = y) \wedge x = y)$$

$$\therefore \quad \exists x \exists y (x = y \wedge \text{BEL}_\alpha(Mx \wedge Ey))$$

1.	$\exists x \exists y (\forall z (Mz \leftrightarrow z = x) \wedge \forall z (Ez \leftrightarrow z = y) \wedge x = y)$	Δ , R2
2.	$\exists x \exists y (Mx \wedge Ey \wedge \text{BEL}_\alpha(Mx \wedge Ey))$	Δ , R1
3.	$\forall z (Mz \leftrightarrow z = a) \wedge \forall z (Ez \leftrightarrow z = b) \wedge a = b$	1, EI
4.	$Mc \wedge Ed \wedge \text{BEL}_\alpha(Mc \wedge Ed)$	2, EI
5.	$\forall z (Mz \leftrightarrow z = a)$	3, s
6.	$Mc \leftrightarrow c = a$	6, UI
7.	Mc	4, s
8.	$c = a$	6,7, MP
9.	$\forall z (Ez \leftrightarrow z = b)$	3, s
10.	$Ed \leftrightarrow d = b$	9, UI
11.	Ed	4, s
12.	$d = b$	10,11, MP
13.	$a = b$	3, s
14.	$c = a \wedge a = b \rightarrow b = c$	Transitivity
15.	$c = a \wedge a = b$	8,13, Adj
16.	$b = c$	14,15, MP
17.	$d = b \wedge b = c \rightarrow c = d$	Transitivity
18.	$d = b \wedge b = c$	12,16, Adj
19.	$c = d$	17,18, MP
20.	$\text{BEL}_\alpha(Mc \wedge Ed)$	4, s
21.	$c = d \wedge \text{BEL}_\alpha(Mc \wedge Ed)$	19,20, Adj
22.	$\exists x \exists y (x = y \wedge \text{BEL}_\alpha(Mx \wedge Ey))$	21, EG

QED

Secondly, the derivation from the reformulated (**R1-R2**) to the absurd conclusion (note that this is not a formal derivation, but a semi-formal, semantic derivation):

R1-R2: $\exists x \exists y (x = y \wedge \text{BEL}_\alpha(Mx \wedge Ey))$

$\therefore \text{BEL}_\alpha(\exists z(Mz \wedge Ez))$

- | | | |
|-----|--|---------------------|
| 1. | $\exists x \exists y (x = y \wedge \text{BEL}_\alpha(Mx \wedge Ey))$ | Δ |
| 2. | $a = b \wedge \text{BEL}_\alpha(Ma \wedge Eb)$ | 1, (E) |
| 3. | $\text{BEL}_\alpha(Ma \wedge Eb)$ | 2, (C) |
| 4. | $\forall w' \in \mathcal{B}_\alpha(Ma \wedge Eb)$ | 3, (B) |
| 5. | $\forall w' \in \mathcal{B}_\alpha(Ma)$ | 4, (C) |
| 6. | $a = b$ | 2, (C) |
| 7. | $a = b \rightarrow \forall w' (a = b)$ | 6, (R) |
| 8. | $\forall w' (a = b)$ | 6, 7, MP |
| 9. | $\forall w' \in \mathcal{B}_\alpha(Mb)$ | 5, 8, (P) |
| 10. | $\forall w' \in \mathcal{B}_\alpha(Eb)$ | 4, (C) |
| 11. | $\forall w' \in \mathcal{B}_\alpha(Mb \wedge Eb)$ | 9, 10, (C) |
| 12. | $\forall w' \in \mathcal{B}_\alpha(\exists z(Mz \wedge Ez))$ | 11, (E) |
| 13. | $\text{BEL}_\alpha(\exists z(Mz \wedge Ez))$ | 12, (B) |

QED